

Study of the $B_c \rightarrow B_s \pi$ decay with the perturbative QCD approach

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Abstract

The $B_c \rightarrow B_s \pi$ decay is studied with the perturbative QCD approach. Three types of wave functions for B_s meson are considered. The transition form factor $F_0^{B_c \rightarrow B_s}(0)$ and the branching ratio $\mathcal{B}r(B_c \rightarrow B_s \pi)$ are sensitive to the model of the B_s meson wave functions. With appropriate inputs, our estimate on $\mathcal{B}r(B_c \rightarrow B_s \pi)$ is comparable with the recent LHCb measurement. A clear signal of $B_c \rightarrow B_s \pi$ decay should be easily observed at the Large Hadron Collider.

Keywords: B_c meson, weak decay, the perturbative QCD approach, branching ratio.

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I. INTRODUCTION

The B_c meson, the heaviest of the ground pseudoscalar meson with explicit both bottom and charm quantum numbers, was observed for the first time via the decay $B_c \rightarrow J/\psi \ell \nu$ in 1.8 TeV $p\bar{p}$ collisions using the CDF detector at the Fermilab Tevatron in 1998 [1]. The mass is currently measured at the $\mathcal{O}(10^{-4})$ level [2, 3]. However, there is about 10% uncertainty in the present B_c lifetime measurement [4, 5].

The B_c meson, laying below BD threshold, is stable for both the strong and electromagnetic annihilation interactions. It can decay only via the weak interaction. The decay modes can be divided into three classes (taking B_c^+ as an illustration) [6, 7]: (1) the c quark decays while the \bar{b} quark as a spectator, that is, $c \rightarrow W^+ + s$ (or d); (2) the \bar{b} quark decays while the c quark as a spectator, that is, $\bar{b} \rightarrow W^+ + \bar{c}$ (or \bar{u}); (3) the annihilation channel, that is, $c + \bar{b} \rightarrow W^+$; where the virtual W^+ boson materializes either into a pair of leptons $\ell^+ \nu$, or into a pair of quarks which then hadronizes. Both the c and \bar{b} quarks in B_c^+ meson can decay individually, resulting in that the B_c lifetime is about three times less than other ground pseudoscalar b -flavored mesons, that is, $\tau_{B_c} < \frac{1}{3} \tau_{B_{u,d,s}}$ [8]. Enhanced by the hierarchy of the Cabibbo-Kobayashi-Maskawa (CKM) quark-mixing matrix elements $|V_{cs}| > |V_{cb}|$, the class (1) decay contributes $\sim 70\%$ to the B_c decay width [9], which shows the critical contribution of the charm quark to the B_c lifetime. In this paper, we study the $B_c \rightarrow B_s \pi$ decay which belongs to class (1). Our motivations are listed below.

Firstly, from an experimental consideration. It is estimated [10] that with large production cross section $\sigma(B_c) \sim 1\mu\text{b}$ (taking into account the cascade decays of the excited states B_c^* below BD threshold) and high luminosity of $\mathcal{L} = 10^{34}\text{cm}^{-2}\text{s}^{-1}$, one could expect 5×10^{10} of B_c meson per year at LHC. In addition, LHCb, a dedicated b -physics precision experiment at LHC, allows proper time resolution of ~ 50 fs [11] which is one order of magnitude shorter than lifetime of the $B_{c,s}$ meson. This property facilitates the separation between primary and decay vertices. The rate for the charged pion identification is larger than 90% [11]. There seems to be a golden opportunity and a real possibility to investigate the B_c weak decay. The reconstructed events of the $B_c \rightarrow B_s \pi$ decay is estimated to be $\sim 10^3$ per year [10]. Recently, the LHCb collaboration has presented research that the $B_c \rightarrow B_s \pi$ decay is observed with significance in excess of five standard deviations based on 3 fb^{-1} data sample [12].

Secondly, from phenomenological considerations. With abundant measurements, we can carefully test various theoretical models. Accordingly, accurate theoretical prediction of B_c decay is essential, although the B_c weak decay is complicated because of strong interaction effects. Based on an expansion in α_s/π and Λ_{QCD}/m_Q (where α_s , Λ_{QCD} and m_Q are the strong coupling constant, QCD characteristic scale and mass of heavy quark Q , respectively), several attractive methods have been proposed to evaluate the hadronic matrix elements, such as the QCD factorization [13], perturbative QCD method (pQCD) [14], and soft and collinear effective theory [15]. These approaches have been applied to $B_{u,d}$ hadronic decays with reasonable explanation for measurements. Whether or not these approach is suitable for $B_c \rightarrow B_s \pi$ decay needs to be examined. Herein we investigate $B_c \rightarrow B_s \pi$ decay with the pQCD approach to give an estimate of the associated branching ratio.

II. EFFECTIVE HAMILTONIAN

For hadronic decays, typically the effective Hamiltonian calculations with operator product expansion scheme is used. The effective Hamiltonian for $B_c \rightarrow B_s \pi$ decay can be written as [16]:

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{ud} V_{cs}^* \left\{ C_1(\mu) (\bar{s}_\alpha c_\alpha)_{V-A} (\bar{u}_\beta d_\beta)_{V-A} + C_2(\mu) (\bar{s}_\alpha c_\beta)_{V-A} (\bar{u}_\alpha d_\beta)_{V-A} \right\} + \text{H.c.}, \quad (1)$$

where G_F is Fermi coupling constant. The CKM factor $V_{ud} V_{cs}^* \sim \mathcal{O}(1)$. α and β are color indices. $(\bar{q}q')_{V-A} = \bar{q} \gamma_\mu (1 - \gamma_5) q'$. The Wilson coefficients $C_{1,2}(\mu)$, incorporating the physics contributions from heavy particles such as W and Z bosons, top quark, and scales higher than μ , have been calculated to the next-to-leading order with perturbation theory [16]. There most difficult problem remaining in calculation is how to evaluate the hadronic matrix elements of the local operators properly and accurately.

III. HADRONIC MATRIX ELEMENTS

Using Brodsky-Lepage approach [17], the hadronic matrix element is commonly written as a convolution of hard-scattering kernels and hadron wave functions (WFs). It is shown that [13] due to appearance of the endpoint divergences, the QCDF formalisms based on the collinear approximation fail to give the contributions of the spectator and annihilation

interactions satisfactorily. The pQCD approach advocates that [14] the endpoint singularity in collinear approximation could be smeared by retaining the transverse momentum k_T of quarks and by introducing the Sudakov factor e^{-S} . Using the pQCD formalisms, one decay amplitude is factorized into three factors: the “harder” effects incorporated into the Wilson coefficients C , the heavy quark decay subamplitude H , and the universal hadron WFs Φ , such that:

$$\int d\mathbf{k} C(t) H(k, t) \Phi(k) e^{-S} \quad (2)$$

where k and t are the corresponding kinematic variables and characteristic scale, respectively.

A. Kinematic variables

In the terms of the light cone coordinate, the momenta of the valence quarks and hadrons in the rest frame of the B_c meson are defined as:

$$p_1 = \frac{m_1}{\sqrt{2}}(1, 1, \vec{0}_T), \quad (3)$$

$$p_2 = \frac{m_1}{\sqrt{2}}(r_{B_s}^2, 1, \vec{0}_T), \quad (4)$$

$$p_3 = \frac{m_1}{\sqrt{2}}(1 - r_{B_s}^2, 0, \vec{0}_T), \quad (5)$$

$$k_i = x_i p_i + (0, 0, \vec{k}_{iT}), \quad (6)$$

where the subscript $i = 1, 2, 3$ refer to B_c , B_s , π meson, respectively. Variables k_i , \vec{k}_{iT} , x_i are the four-dimensional momentum, transverse momentum and longitudinal momentum fraction of light quark, respectively. $r_{B_s} = m_{B_s}/m_{B_c}$.

B. Wave functions

Hadron wave functions are basic input in Eq.(2). Adopting the notation in [18–27], the two-valence-particle WFs for double-light pion, heavy-light B_s meson and double-heavy B_c meson are decomposed into :

$$\langle \pi(p_3) | \bar{u}_\alpha(z) d_\beta(0) | 0 \rangle = \frac{-i}{\sqrt{2N_c}} \int d^4 k_3 e^{+ik_3 \cdot z} \left\{ \gamma_5 \left[\not{p}_3 \phi_\pi^a + \mu_\pi \phi_\pi^p + \mu_\pi (\not{n}_+ \not{n}_- - 1) \phi_\pi^t \right] \right\}_{\beta\alpha}, \quad (7)$$

$$\langle B_s(p_2) | \bar{s}_\alpha(z) b_\beta(0) | 0 \rangle = \frac{-i}{\sqrt{2N_c}} \int d^4 k_2 e^{+ik_2 \cdot z} \left\{ \gamma_5 \left(\not{p}_2 + m_{B_s} \right) \left[\frac{\not{n}_+}{\sqrt{2}} \phi_{B_s}^- + \frac{\not{n}_-}{\sqrt{2}} \phi_{B_s}^+ \right] \right\}_{\beta\alpha}, \quad (8)$$

$$\langle 0 | \bar{b}_\alpha(0) c_\beta(z) | B_c(p_1) \rangle = \frac{-i}{\sqrt{2N_c}} \int d^4 k_1 e^{-ik_1 \cdot z} \left\{ \left[\frac{\not{n}_+}{\sqrt{2}} \phi_{B_c}^+ + \frac{\not{n}_-}{\sqrt{2}} \phi_{B_c}^- \right] (\not{p}_1 + m_{B_c}) \gamma_5 \right\}_{\beta\alpha}, \quad (9)$$

where $N_c = 3$ is the color number. n_- and n_+ are the minus and plus null vectors, respectively. $n_+ \cdot n_- = 1$.

For the heavy-light B_s meson, there are two scalar WFs $\phi_{B_s}^+$ and $\phi_{B_s}^-$. The equation of motion for WFs $\phi_{B_s}^\pm$ is [18, 19]

$$\phi_{B_s}^+(x) + x \phi_{B_s}^{-'}(x) = 0. \quad (10)$$

The relationship is helpful in constraining models for the B_s WFs, which leads to $\phi_{B_s}^+(x)$ vanished at the endpoint and $\phi_{B_s}^-(x) = \mathcal{O}(1)$ for $x \rightarrow 0$ [20]. Here we will investigate three candidates of WFs for B_s meson.

The first candidate is the exponential type (GN) suggested in [21],

$$\phi_{B_s}^+(x, b) = \frac{f_{B_s}}{2\sqrt{2N_c}} N_{\text{GN}}^+ x \exp\left[-\frac{x m_{B_s}}{\omega_{\text{GN}}}\right] \frac{1}{1 + (b \omega_{\text{GN}})^2}, \quad (11)$$

$$\phi_{B_s}^-(x, b) = \frac{f_{B_s}}{2\sqrt{2N_c}} N_{\text{GN}}^- \exp\left[-\frac{x m_{B_s}}{\omega_{\text{GN}}}\right] \frac{1}{1 + (b \omega_{\text{GN}})^2}. \quad (12)$$

The second candidate is the Gaussian type (KLS) proposed in [22, 23] whereby

$$\phi_{B_s}^+(x, b) = \frac{f_{B_s}}{2\sqrt{2N_c}} N_{\text{KLS}}^+ x^2 \bar{x}^2 \exp\left[-\frac{1}{2}\left(\frac{x m_{B_s}}{\omega_{\text{KLS}}}\right)^2 - \frac{1}{2}\omega_{\text{KLS}}^2 b^2\right], \quad (13)$$

$$\begin{aligned} \phi_{B_s}^-(x, b) = & \frac{f_{B_s}}{2\sqrt{2N_c}} N_{\text{KLS}}^- \exp\left[-\frac{1}{2}\omega_{\text{KLS}}^2 b^2\right] \left\{ \exp\left[-\frac{1}{2}\left(\frac{x m_{B_s}}{\omega_{\text{KLS}}}\right)^2\right] (m_{B_s}^2 \bar{x}^2 + 2\omega_{\text{KLS}}^2) \right. \\ & \left. + \sqrt{2\pi} m_{B_s} \omega_{\text{KLS}} \text{Erf}\left(\frac{x m_{B_s}}{\sqrt{2}\omega_{\text{KLS}}}\right) + C_{\text{KLS}} \right\}, \end{aligned} \quad (14)$$

where $\bar{x} = 1 - x$, and the constant C_{KLS} is chosen so that $\phi_{B_s}^-(1, b) = 0$.

The third candidate is the KKQT type derived from the QCD equations of motion and heavy-quark symmetry constraint [19, 24],

$$\phi_{B_s}^+(x, b) = \frac{f_{B_s}}{2\sqrt{2N_c}} \frac{2x}{\omega_{\text{KKQT}}^2} \theta(y) J_0(m_{B_s} b \sqrt{xy}), \quad (15)$$

$$\phi_{B_s}^-(x, b) = \frac{f_{B_s}}{2\sqrt{2N_c}} \frac{2y}{\omega_{\text{KKQT}}^2} \theta(y) J_0(m_{B_s} b \sqrt{xy}), \quad (16)$$

where $y = \omega_{\text{KKQT}} - x$.

In the above equations Eq.(11—16), b denotes the conjugate variables of the transverse momentum of s quark in B_s meson. There is only one parameter ω for each kind of WFs candidate. The normalization constants N^\pm is related to the decay constant f_{B_s} through the relation :

$$\int_0^1 \phi_{B_s}^\pm(x, 0) dx = \frac{f_{B_s}}{2\sqrt{2N_c}}. \quad (17)$$

For the double-light pion, the expression of $\phi_\pi^{a,p,t}$ can be found in [22–26]. For the double-heavy B_c meson, it can be described approximatively by nonrelativistic dynamics. At tree level and in leading order of the expansion in the relative velocities, b and c quarks in the B_c meson just share the total momentum according to their masses [27],

$$\phi_{B_c}^\pm(x) = \frac{f_{B_c}}{2\sqrt{2N_c}}\delta(x - r_c), \quad (18)$$

where f_{B_c} is the decay constant, and $r_c = m_c/m_{B_c}$.

IV. $B_c \rightarrow B_s$ TRANSITION FORM FACTORS

The $B_c \rightarrow B_s$ transition form factors are defined as follows [28]:

$$\begin{aligned} & \langle B_s(p_2) | \bar{s}\gamma^\mu(1 - \gamma_5)c | B_c(p_1) \rangle \\ &= \left[(p_1 + p_2)^\mu - \frac{m_{B_c}^2 - m_{B_s}^2}{q^2} q^\mu \right] F_1^{B_c \rightarrow B_s}(q^2) \\ &+ \frac{m_{B_c}^2 - m_{B_s}^2}{q^2} q^\mu F_0^{B_c \rightarrow B_s}(q^2), \end{aligned} \quad (19)$$

where $q = p_1 - p_2$ is the momentum transfer. Usually, the longitudinal form factor $F_0(q^2)$ is compulsorily equal to the transverse form factor $F_1(q^2)$ in the largest recoil limit to cancel singularities appearing at the pole $q^2 = 0$, i.e., $F_0(0) = F_1(0)$.

Within the pQCD framework, the one-gluon-exchange diagrams contributing to the $B_c \rightarrow B_s$ transition form factors are displayed in Fig.1. It has been shown that the pQCD approach works ideally in the large recoil region [14, 20, 23, 24]. The expression of form factors is :

$$\begin{aligned} & F_0^{B_c \rightarrow B_s}(0) = F_1^{B_c \rightarrow B_s}(0) \\ &= 8\pi m_{B_c}^2 C_F \int_0^1 \mathbf{d}x_1 \int_0^1 \mathbf{d}x_2 \int_0^\infty b_2 \mathbf{d}b_2 \\ &\times \left\{ E_a(t_a) \alpha_s(t_a) H_a(\alpha, \beta_a, b_2) r_{B_s} \phi_{B_s}^+(x_2) \right. \\ &\quad \times \left[\phi_{B_c}^+(x_1) (x_2 r_{B_s} + r_c - x_2) \right. \\ &\quad \left. \left. + \phi_{B_c}^-(x_1) (x_2 r_{B_s} + r_c - r_{B_s} r_c) \right] \right. \\ &\quad \left. + E_b(t_b) \alpha_s(t_b) H_b(\alpha, \beta_b, b_2) x_1 \phi_{B_c}^-(x_1) \right. \\ &\quad \left. \times \left[\phi_{B_s}^-(x_2) (1 - r_{B_s}) + \phi_{B_s}^+(x_2) r_{B_s}^2 \right] \right\}, \end{aligned} \quad (20)$$

where $C_F = 4/3$ is color factor. $E_i(t_i) = e^{-S_{B_s}(t_i)}$ is the Sudakov factor.

$$S_{B_s}(t) = s(x_2 p_2^-, b_2) + 2 \int_{1/b_2}^t \frac{d\mu}{\mu} \gamma_q, \quad (21)$$

where $\gamma_q = -\alpha_s/\pi$ is the quark anomalous dimension. The expression of $s(Q, b)$ can be found in [29, 30]. The hard-scattering kernel function $H_{a,b}$ in Eq.(20) is defined as :

$$H_a(\alpha, \beta_a, b_2) = \frac{K_0(\sqrt{\alpha}b_2) - K_0(\sqrt{\beta}b_2)}{\beta_a - \alpha}, \quad (22)$$

$$H_b(\alpha, \beta_b, b_2) = \frac{K_0(\sqrt{\alpha}b_2)}{\beta_b}, \quad (23)$$

where K_0 and I_0 are the modified Bessel functions; α and β are the virtualities of internal gluons and quarks, respectively.

$$\alpha = -m_{B_c}^2 [\bar{x}_1^2 + r_{B_s}^2 \bar{x}_2^2 - \bar{x}_1 \bar{x}_2 (1 + r_{B_s}^2)], \quad (24)$$

$$\beta_a = -m_{B_c}^2 [1 + r_{B_s}^2 \bar{x}_2^2 - \bar{x}_2 (1 + r_{B_s}^2) - r_c^2], \quad (25)$$

$$\beta_b = -m_{B_c}^2 [r_{B_s}^2 + \bar{x}_1^2 - \bar{x}_1 (1 + r_{B_s}^2)], \quad (26)$$

$$t_i = \max(\sqrt{|\alpha|}, \sqrt{|\beta_i|}, 1/b_2). \quad (27)$$

V. DECAY AMPLITUDE

Within the pQCD framework, the Feynman diagrams for $B_c \rightarrow B_s \pi$ decay are shown in Fig.2, where (a) and (b) are factorizable topology, (c) and (d) are nonfactorizable topology. After a straightforward calculation, the decay amplitudes is written as :

$$\mathcal{A}(B_c \rightarrow B_s \pi) = \frac{G_F}{\sqrt{2}} V_{ud} V_{cs}^* \sum_{i=a,b,c,d} \mathcal{A}_i, \quad (28)$$

where the explicit expressions of \mathcal{A}_i are collected in APPENDIX A. From these expressions, we see that only the twist-2 light-cone distribution amplitude (LCDA) ϕ_π^a of pion contributes to nonfactorizable decay amplitude. The expression of the twist-2 pion LCDA is defined in Gegenbauer polynomials [25] as:

$$\phi_\pi^a(x, \mu) = \frac{f_\pi}{2\sqrt{2N_c}} 6x\bar{x} \{1 + a_2^\pi(\mu) C_2^{3/2}(x - \bar{x}) + a_4^\pi(\mu) C_4^{3/2}(x - \bar{x})\}, \quad (29)$$

where the nonperturbative parameter $a_i^\pi(\mu)$ is the Gegenbauer moment.

The branching ratio in the B_c meson rest frame can be written as:

$$\mathcal{B}r(B_c \rightarrow B_s \pi) = \frac{\tau_{B_c}}{8\pi} \frac{p}{m_{B_c}^2} |\mathcal{A}(B_c \rightarrow B_s \pi)|^2, \quad (30)$$

where the common momentum $p = (m_{B_c}^2 - m_{B_s}^2)/2m_{B_c}$.

VI. NUMERICAL RESULTS AND DISCUSSION

The input parameters in our numerical calculation are collected in Table 1. If not specified explicitly, we shall take their central values as the default input.

Our results on form factor $F_0^{B_c \rightarrow B_s}(0)$ and branching ratio $\mathcal{Br}(B_c \rightarrow B_s \pi)$ are listed in Table 2, where the first uncertainty comes from the mass of charm quark; the second uncertainty comes from the shape parameter of B_s meson WFs, that is, $\omega_{\text{GN}} = 0.65 \pm 0.10$ GeV in Eq.(11-12), $\omega_{\text{KLS}} = 0.75 \pm 0.10$ GeV in Eq.(13-14) and $\omega_{\text{KKQT}} = 0.35 \pm 0.10$ in Eq.(15-16); the third uncertainty comes from the choice of hard scales $(1 \pm 0.1)t_i$ in Eq.(27). We can see that the large uncertainty come from the B_s WFs. In addition, the decay constants f_{B_c} and f_{B_s} will bring some 2.4% uncertainty to the form factor.

For the form factor $F_0^{B_c \rightarrow B_s}(0)$, our study show that : Firstly, the interference between Fig.1(a) and (b) is constructive. Compared with Fig.1(b), the contribution of Fig.1(a) is $\lesssim 30\%$; Secondly, both $\phi_{B_c}^+$ and $\phi_{B_c}^-$ contribute to the form factor, and their interference is constructive. Compared with $\phi_{B_c}^-$, the contribution of $\phi_{B_c}^+$ is $\lesssim 20\%$; Thirdly, the interference between $\phi_{B_s}^+$ and $\phi_{B_s}^-$ is constructive. Compared with $\phi_{B_s}^+$, the contribution of $\phi_{B_s}^-$ is $< 20\%$ for both GN and KLS type; Fourthly, the form factor $F_0^{B_c \rightarrow B_s}(0)$ is sensitive to the shape parameter of B_s WFs; Fifthly, By keeping the parton transverse momentum k_T , and employing the Sudakov factors to suppress the long distance contribution in large b region [14], the form factor $F_0^{B_c \rightarrow B_s}(0)$ is perturbatively calculable with the pQCD approach. The contribution to form factor comes completely from $\alpha_s/\pi < 0.3$ region with the scale of Eq.(27); Lastly, considering the uncertainties, our results is reasonable agreement with the previous results listed in Table 3.

For the branching ratio $\mathcal{Br}(B_c \rightarrow B_s \pi)$, our study show that : Firstly, the dominated contribution comes from the factorizable topology Fig.2(a,b). The interference between nonfactorizable diagrams Fig.2(c) and (d) is destructive; Secondly, the main uncertainty is from B_s WFs. Considering the uncertainties, our results is basically comparable with the previous results listed in Table 3, and is also agreement with recent LHCb estimate $\mathcal{Br}(B_c \rightarrow B_s \pi) \sim 10\%$ [12].

VII. SUMMARY

Herein we consider three models (exponential, Gaussian, and KKQT type) of B_s meson WFs, and study the $B_c \rightarrow B_s$ transition form factor and the branching ratio for the $B_c \rightarrow B_s \pi$ decay with the pQCD approach. We find that (1) By keeping the parton transverse momentum k_T and employing the Sudakov factors, $F_0^{B_c \rightarrow B_s}(0)$ and $\mathcal{B}r(B_c \rightarrow B_s \pi)$ are perturbatively calculable within the pQCD framework. The contribution to form factor comes completely from $\alpha_s/\pi < 0.3$ region. (2) $F_0^{B_c \rightarrow B_s}(0)$ and $\mathcal{B}r(B_c \rightarrow B_s \pi)$ are sensitive to the model and the shape parameter ω of the B_s meson wave functions. (3) With appropriate inputs, our estimates on $F_0^{B_c \rightarrow B_s}(0)$ and $\mathcal{B}r(B_c \rightarrow B_s \pi)$ are comparable with the previous results. The branching ratio for the $B_c \rightarrow B_s \pi$ decay is about a few per cent, which is agreement with the recent LHCb measurement.

With the running of LHC, the more data will be accumulated at LHCb, the more precise branching ratio for the $B_c \rightarrow B_s \pi$ decay will be obtained, and various models for hadron wave function and factorization treatments for hadron matrix element will be more stringently tested.

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Appendix A: the decay amplitudes

There are four diagrams contributing to the $B_c \rightarrow B_s \pi$ decay which are shown in Fig.2. The expression of the decay amplitudes in Eq.(28) are :

$$\begin{aligned}
\mathcal{A}_a &= -i8\pi C_F f_\pi m_{B_c}^4 (1 - r_{B_s}^2) r_{B_s} \int_0^1 \mathbf{d}x_1 \mathbf{d}x_2 \int_0^\infty b_2 \mathbf{d}b_2 \\
&\times E_a(t_a) \alpha_s(t_a) H_a(\alpha, \beta_a, b_2) \left[C_1(t_a) + \frac{1}{N_c} C_2(t_a) \right] \phi_{B_s}^+(x_2) \\
&\times \left\{ \phi_{B_c}^+(x_1) [x_2 r_{B_s} + r_c - x_2] + \phi_{B_c}^-(x_1) [x_2 r_{B_s} + r_c - r_{B_s} r_c] \right\}, \\
\mathcal{A}_b &= -i8\pi C_F f_\pi m_{B_c}^4 (1 - r_{B_s}^2) \int_0^1 \mathbf{d}x_1 \mathbf{d}x_2 \int_0^\infty b_2 \mathbf{d}b_2
\end{aligned} \tag{A1}$$

$$\begin{aligned} & \times E_b(t_b)\alpha_s(t_b)H_b(\alpha, \beta_b, b_2)\left[C_1(t_b) + \frac{1}{N_c}C_2(t_b)\right]x_1\phi_{B_c}^-(x_1) \\ & \times \left\{\phi_{B_s}^-(x_2)\left[1 - r_{B_s}\right] + \phi_{B_s}^+(x_2)r_{B_s}^2\right\}, \end{aligned} \quad (\text{A2})$$

$$\begin{aligned} \mathcal{A}_c &= \frac{-i32\pi C_F m_{B_c}^4}{\sqrt{2N_c}}(1 - r_{B_s}^2)r_{B_s}\int_0^1 \mathbf{d}x_1 \mathbf{d}x_2 \mathbf{d}x_3 \int_0^\infty b_2 \mathbf{d}b_2 b_3 \mathbf{d}b_3 \\ & \times E_c(t_c)\alpha_s(t_c)H_c(\alpha, \beta_c, b_2, b_3)C_2(t_c)\phi_{B_s}^+(x_2)\phi_\pi^a(x_3) \\ & \times \left\{\phi_{B_c}^+(x_1)(1 - r_{B_s})(x_2 - x_1) + \phi_{B_c}^-(x_1)\left[r_{B_s}^2(x_2 - x_3) + (x_3 - x_1)\right]\right\}, \end{aligned} \quad (\text{A3})$$

$$\begin{aligned} \mathcal{A}_d &= \frac{-i32\pi C_F m_{B_c}^4}{\sqrt{2N_c}}(1 - r_{B_s}^2)\int_0^1 \mathbf{d}x_1 \mathbf{d}x_2 \mathbf{d}x_3 \int_0^\infty b_2 \mathbf{d}b_2 b_3 \mathbf{d}b_3 \\ & \times E_d(t_d)\alpha_s(t_d)H_d(\alpha, \beta_d, b_2, b_3)C_2(t_d)\phi_{B_c}^-(x_1)\phi_\pi^a(x_3) \\ & \times \left\{\phi_{B_s}^+(x_2)r_{B_s}\left[(x_1 - \bar{x}_3) - r_{B_s}^2(x_2 - \bar{x}_3)\right] - \phi_{B_s}^-(x_2)(1 - r_{B_s})(x_1 - x_2)\right\}, \end{aligned} \quad (\text{A4})$$

where the evolution factor $E_{a,b}$ and the kernel function $H_{a,b}$ are the same as those for the $B_c \rightarrow B_s$ transition form factors given in Section IV. The evolution factor $E_{c,d} = e^{-S_{B_s}-S_\pi}$. The Sudakov factors S_{B_s} are given in Eq.(21). And the Sudakov factors S_π is defined as :

$$S_\pi(t) = s(x_3 p_3^+, b_3) + s(\bar{x}_3 p_3^+, b_3) + 2 \int_{1/b_3}^t \frac{\mathbf{d}\mu}{\mu} \gamma_q. \quad (\text{A5})$$

The hard-scattering kernel function $H_{c,d}$ is defined as :

$$H_i(\alpha, \beta_i, b_2, b_3) = K_0(\sqrt{\beta_i b_3}) \left\{ \theta(b_2 - b_3) K_0(b_2 \sqrt{\alpha}) I_0(b_3 \sqrt{\alpha}) + b_2 \leftrightarrow b_3 \right\}, \quad (\text{A6})$$

where α and β_i are the virtualities of internal gluons and quarks in Fig.2, respectively; t_i is the maximal (either longitudinal or transverse) virtuality of propagators:

$$\beta_c = -m_1^2(x_1 - x_2) \left[x_1 - x_3 - r_{B_s}^2(x_2 - x_3) \right], \quad (\text{A7})$$

$$\beta_d = -m_1^2(x_1 - x_2) \left[x_1 - \bar{x}_3 - r_{B_s}^2(x_2 - \bar{x}_3) \right], \quad (\text{A8})$$

$$t_i = \max(\sqrt{|\alpha|}, \sqrt{|\beta_i|}, 1/b_2, 1/b_3) \quad \text{for } i = c, d. \quad (\text{A9})$$

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Table 1: input parameters for $B_c \rightarrow B_s \pi$ decay

parameter	numerical value	Ref.
mass of B_c	$m_{B_c} = 6.277 \pm 0.006 \text{ GeV}$	[8]
mass of B_s	$m_{B_s} = 5366.7 \pm 0.4 \text{ MeV}$	[8]
mass of c quark	$m_c = 1.275 \pm 0.025 \text{ GeV}$	[8]
lifetime of B_c	$\tau_{B_c} = 0.453 \pm 0.041 \text{ ps}$	[8]
decay constant of π	$f_\pi = 130.41 \pm 0.03 \pm 0.20 \text{ MeV}$	[8]
decay constant of B_c	$f_{B_c} = 489 \pm 4 \pm 3 \text{ MeV}$	[31]
decay constant of B_s	$f_{B_s} = 227.6 \pm 5.0 \text{ MeV}$	[32]
Gegenbauer moment	$a_2^\pi(1\text{GeV}) = 0.16 \pm 0.01$	[33]
	$a_4^\pi(1\text{GeV}) = 0.04 \pm 0.01$	[33]

Table 2: Form factor $F_0^{B_c \rightarrow B_s}(0)$ and branching ratio $\mathcal{B}r(B_c \rightarrow B_s \pi)$.

	GN	KLS	KKQT
F_0	$0.84^{+0.06+0.28+0.09}_{-0.05-0.20-0.05}$	$0.87^{+0.08+0.36+0.09}_{-0.07-0.25-0.05}$	$0.62^{+0.06+1.08+0.06}_{-0.05-0.37-0.04}$
$\mathcal{B}r \times 10^2$	$6.20^{+0.90+4.78+1.83}_{-0.75-2.60-0.90}$	$6.57^{+1.19+6.57+1.90}_{-0.98-3.26-0.94}$	$3.40^{+0.65+22.32+0.79}_{-0.52-2.84-0.48}$

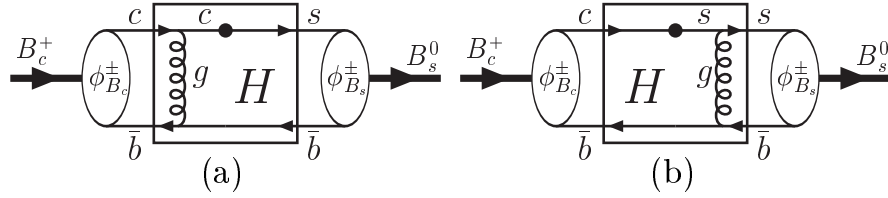


Fig. 1: Diagrams contributing to the $B_c \rightarrow B_s$ transition form factor, where the box represents the lowest order hard-scattering kernel H , and the dot denotes an appropriate Dirac matrix.

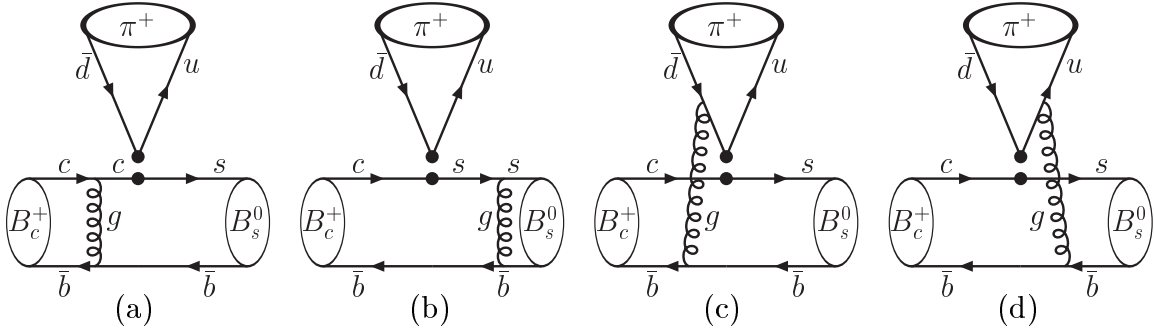


Fig. 2: Feynman diagrams for $B_c \rightarrow B_s \pi$ decay within the pQCD framework, where the dots denote an appropriate Dirac matrix.

Table 3: Form factor $F_0^{B_c \rightarrow B_s}(0)$ and branching ratio $\mathcal{B}r(B_c \rightarrow B_s \pi)$ (in the unit of %) in the previous literature.

Ref.	F_0	Ref.	F_0	$\mathcal{B}r \times 10^2$	Ref.	$\mathcal{B}r \times 10^2$
[34]	0.926^a	[35]	1.03	$12.01 [10.9]^b$	[37]	5.31^c
[38]	1.021^d	[39]	$0.573 [0.571]^e$	$3.723 [3.697]^e$	[40]	3.9^f
[41]	$0.73^{+0.03+0.03g}_{-0.04-0.03}$	[42]	0.55 ± 0.03^h	4.8 ± 0.5^h	[43]	1.57^i
[44]	1.02^j	[45]	$0.58^{+0.01k}_{-0.02}$	$3.51^{+0.19k}_{-0.06}$	[46]	4.30^i
[47]	0.297^a	[48]	0.50^f	2.52^f	[49]	5.5^l
[50]	0.61^f	[51]	1.3^m	16.4^m	[7]	5.93
[52]	$0.403 \sim 0.617^n$	[53]	1.3^m	17.5^m	[6]	2.19^h
[54]	0.60 ± 0.12^o	[55]	0.66^p	4.0^p	[6]	3.10^a
[56]	0.30 ± 0.05^m	[57]	0.61	3.28^h	[57]	4.66^a
[58]	0.5917^f	[59]	$0.340 \sim 0.925^h$	$1.23 \sim 9.09^h$		
		[60]	0.564^a	4.37^a		

^aIt is estimated with the Isgur-Scora-Grinstein-Wise [36] quark model.

^bIt is estimated with the relativistic quark model, where $a_1 = 1.26$ [1.2].

^cIt is estimated with the QCD factorization approach [13] at the leading order.

^dIt is estimated with a relativistic quark model based on a confining potential in the equally mixed scalar-vector harmonic form.

^eIt is estimated with the light-front quark model, where the interaction is Coulomb plus linear [harmonic oscillator] confining potentials.

^fIt is estimated with a relativistic quark model.

^gIt is estimated with covariant light-front quark model, where the uncertainties are from the decay constant of B_c and B_s mesons.

^hIt is estimated with the BSW [28] model.

ⁱIt is estimated with a relativistic quark model based on the Bethe-Salpeter equation, where $a_1 = 1.2$.

^jIt is estimated with the light-cone QCD sum rules.

^kIt is estimated with the framework of a nonrelativistic constituent quark model.

^lIt is estimated with covariant quark model.

^mIt is estimated with QCD sum rules.

ⁿIt is estimated with the BSW [28] formalism, where uncertainty is from mass of b , c , s quark and shape parameter ω of wave functions.

^oIt is estimated by vertex sum rules.

^pIt is estimated by overlap integral of the meson wave functions.